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13.2: Numerical Modeling of Cavities with Low External Q

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Abstract: *We describe a new simulation technique to find the resonant frequencies and Qs of cavity eigenmodes when the cavity is coupled to an external waveguide. The method treats the cavity first as a closed system to find a set of lossless eigenmodes, then couples this system to a representation of the external waveguide to determine the coupled mode frequency and* $Q_{external}$. *There is no assumption of weak coupling, and the technique is ideal for the analysis of very low-Q output cavities for rf amplifiers.*

Keywords: electromagnetic simulation; broadband cavity; klystron simulation.

Introduction

Large-signal codes for modeling klystrons and multiple-beam klystrons, such as TESLA, typically use resonant eigenmodes to represent the cavity fields [1]. For broadband input and output circuits, that have low $Q_{external}$, a traditional cavity eigenmode representation is more difficult to define due to the strong coupling of the cavity to an external waveguide. Usually such cavities are analyzed using 3D electromagnetic solvers with the addition of an absorbing boundary condition inside the waveguide port to determine a lossy eigenmode of the open system, or one must resort to computing many driven-frequency solutions across the frequency band and perform subsequent field analysis to determine the cavity stored energy dependence on frequency, and hence extract the resonance frequency and Q. The analysis becomes more problematic when multiple overlapping resonance peaks occur inside the operating bandwidth, as is often the case for broadband amplifiers. In the present approach, we avoid the need for eigenmode analysis of the open cavity-waveguide system, and introduce a new technique using only modes of a finite closed cavity, terminated at the waveguide aperture by short-circuiting either the tangential electric or magnetic field. By representing fields inside the cavity using a dual set of lossless eigenmodes we can subsequently reintroduce coupling to the external waveguide to reproduce the actual frequency and $Q_{external}$ of the open cavity-waveguide system. We present the theory behind the analysis, and results from simulations that demonstrate the efficiency of the algorithm.

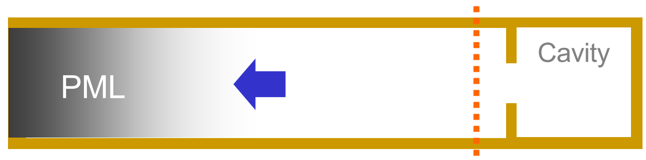


Figure 1. 2D cavity coupled to external waveguide. The dotted line signifies an arbitrary simulation boundary: to the left, fields are described by a 1D equation for the fundamental TE10 mode amplitude, with a PML absorbing boundary condition; to the right, fields are represented using a computed set of basis fields.

Numerical Method

A simple 2D geometry representing a cavity coupled to a waveguide, through an iris, is shown in Figure 1. We close the cavity at an arbitrary simulation boundary inside the waveguide, denoted by the dotted line, and analyze first the eigenmodes of the closed cavity.

Eigenmodes of a closed cavity represent solutions (E_n, ω_n) of the following system of equations for the electric field,

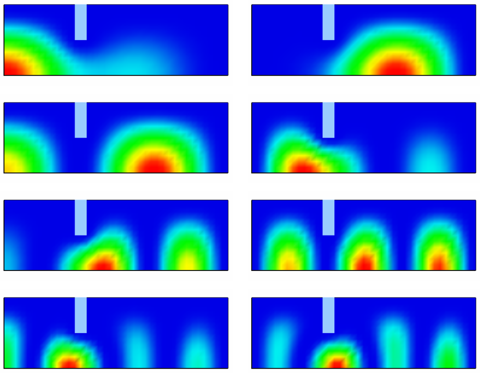
$$\vec{\nabla}\times(\mu^{-1}\vec{\nabla}\times\vec{E})-\omega^2\varepsilon\,\vec{E}=0 \qquad \text{(Vacuum)}$$

$$\vec{n}\times\vec{E}=0 \qquad \text{(Metal boundary)}$$

$$\vec{n}\times(\mu^{-1}\vec{\nabla}\times\vec{E})=0 \qquad \text{(Symmetry boundary)}$$

where ε and μ represent the permittivity and permeability distributions respectively and n is the outward normal at the boundary. For our purposes, we first introduce a symmetry boundary condition at the closed interface and solve for a small set of the lowest frequency eigenmodes of the closed cavity. We then instead introduce a metal boundary condition at the same interface, and solve again for a small set of the lowest frequency eigenmodes. Figure 2(a) shows the two sets of eigenmodes for the present example. These eight modes will be used as a basis set to represent the cavity fields when we solve for the externally coupled system.

(a)



(b)

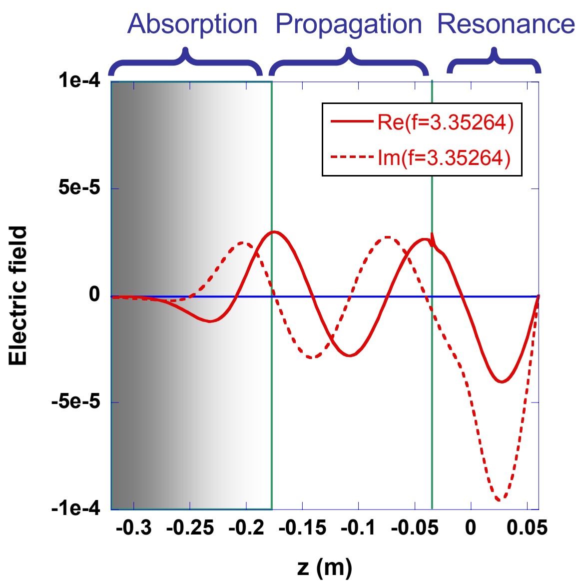


Figure 2. (a) Electric field profiles of basis modes: four (to the left) computed with a magnetically shorted aperture, four (to the right) computed with an electrically shorted aperture. (b) Leaky eigenmode solution of the coupled system of equations (1D and 2D representations) showing resonance, propagation and absorption in the cavity (2D), waveguide (1D) and PML (1D) regions.

In the external waveguide, the fields may be described in terms of transverse modes of the waveguide cross-section. We use a Telegraphists's equation representation for a single transverse mode to reduce the waveguide fields to a discrete 1D model, with a Perfectly Matched Layer (PML) to absorb outgoing waves across a broad range of frequencies. Finally, we couple the cavity and waveguide representations together by exchanging surface currents associated with the fields on either side of the common interface.

This method allows us to obtain a reduced matrix eigenvalue equation for the coupled system, which may be solved using standard numerical techniques. Since the cavity is now externally coupled, eigenmodes of the combined system are lossy modes, with a $Q_{external}$ describing the characteristic loss rate.

Figure 2(b) shows the field profile for a lossy eigenmode of the externally coupled cavity, reconstructed using the new technique, having frequency 3.3526 GHz and a $Q_{external}$ of 58.4. These values agree very well with a conventional electromagnetic solution of the same geometry (3.3523 GHz, $Q_{external}$ = 57.97). The complex-field profile exhibits excellent continuity across the simulation boundary, showing standing-wave character in the cavity, propagation in the waveguide and absorption in the PML.

Conclusion

We have demonstrated a new method for analyzing the lossy eigenmodes of cavities coupled to external waveguides. The method is based on straightforward analysis of the much simpler closed-cavity problem, with external coupling computed via a subsequent analysis. The method is applicable to 3D as well as 2D cavities, and is potentially attractive for modeling complex 3D low-Q cavities, such as are often used in the output cavities of broadband klystron amplifiers.

Acknowledgements

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Reference

1. Igor A. Chernyavskiy et al. "Large-signal code TESLA: current status and recent development," 9th IEEE International Vacuum Electronics Conference (IVEC08), Monterey, CA, 2008. Presentation 13.1.